Geospatial UK

Newcastle University

Where on Earth?

This activity demonstrates some of the mathematical techniques used by Geospatial Engineers when using GPS positioning from satellites

20 - 30 mins

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Introduction

This practical will guide students through mathematical problems Geospatial Engineers may face when using trilateration to locate a GPS receiver using satellite data.

What do we use GPS for?

Global Positioning System (GPS) is type of Global Navigation Satellite System (GNSS) made up of at least 24 satellites. GPS works in any weather conditions, anywhere in the world, 24 hours a day.

We use GPS data in our everyday lives, including to check where our nearest café is, tracking deliveries and even catching Pokémon on the go!

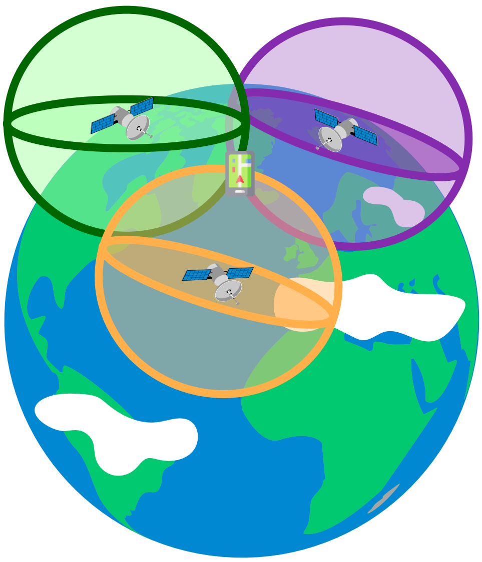
Most GPS-enabled smartphones are typically accurate to within 5 metres, however sometimes we need more precise measurements, down to millimetre scale.

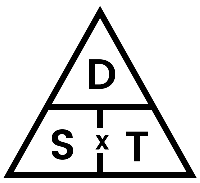
Geospatial Engineers use very precise location data to help make decisions which benefit the environment, for example, topographic mapping to determine where to build a new block of flats, deciding where to build a new wind farm, or even monitoring uplift on volcanoes which might indicate an eruption.

How GPS positioning works

GPS satellites circle the Earth twice a day in a precise orbit. Each satellite transmits a unique signal that allows GPS receivers (e.g., smartphone, GPS handset, etc.) to compute the precise location of the satellite. GPS receivers then use this information to calculate the exact location of the receiver on Earth.

GPS positioning is based on two mathematical concepts – the first is the relationship between **speed, distance, and time**, and the second is called **Trilateration** which is the positioning from 3 (or more) distances.





In this exercise, you will simulate GPS using mathematical concepts.

Learning Outcomes

In this practical, students will learn how to:

- Convert time to distance using the relationship *Distance = Speed × Time*
- Use Standard Form
- Calculate speed using percentages
- Use a given scale to convert real life distances to map distances
- Locate points using coordinates
- Use loci and constructions to locate an unknown point

This activity is aimed at KS4 students; however, students of all ages are welcome to complete the activity.

Contents

Section 1: Satellite Signals

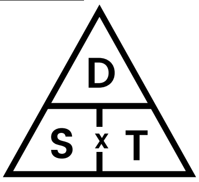
In this exercise, you are going to calculate the exact position of the GPS receiver using the position of 4 satellites - called A, B, C and D.

Each satellite emits a unique signal and records the time it was sent at, then the receiver records what time it receives the signal. Now the receiver can calculate the difference between the time the signal was sent from the satellite and time it was received.

The table below states the time taken for the signal from each satellite to reach the receiver.

	Satellite A	Satellite B	Satellite C	Satellite D
Time for signal to reach receiver in seconds	7.72×10^{-5}	1.03×10^{-4}	8.65×10^{-5}	1.33×10^{-4}

In theory, signals travel at the speed of light, which is $3.0 \times 10^{8}\ m/s$.



1. Using the relationship $Distance = Speed\ \times Time,$ calculate the distance in metres between each satellite and the receiver.

Satellite A: ______________m

Satellite B: ______________m

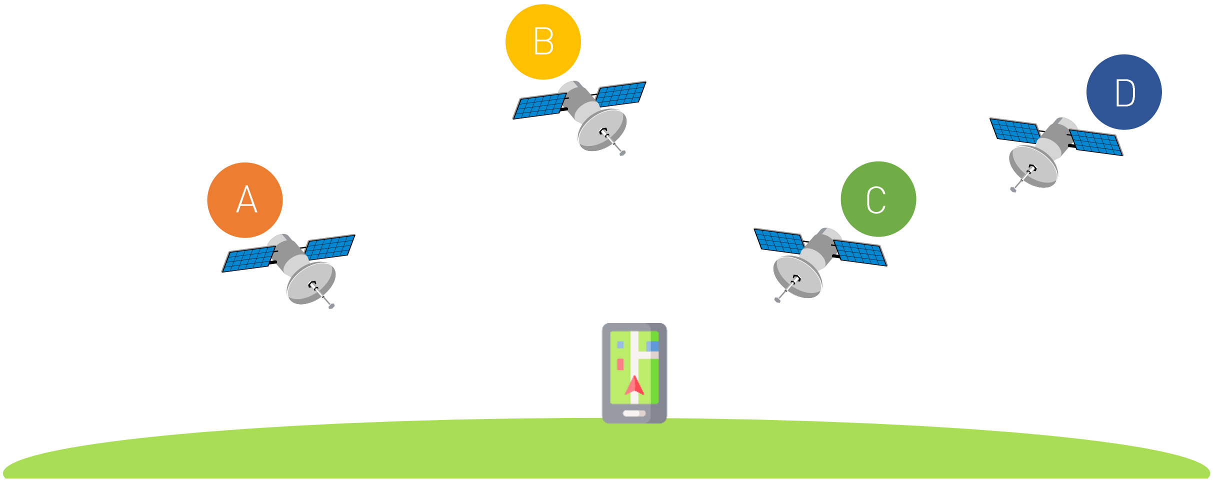
Satellite C: ______________m

Satellite D: ______________m

Section 2: Atmospheric Effects

There are a number of errors which can affect the speed of the signal being transmitted from the satellite to the GPS receiver, including Earth's atmosphere.

In the Troposphere, the lowest part of the atmosphere, there is an abundance of water vapour. The water vapour refracts the signal causing it to arrive later than expected. This is known as **tropospheric delay**.



Shown above is a general layout of the satellites in space. Information is given below about the tropospheric delay for each satellite.

	Satellite A	Satellite B	Satellite C	Satellite D
Time for signal to reach receiver in seconds	7.72×10^{-5}	1.03×10^{-4}	8.65×10^{-5}	1.33×10^{-4}
Tropospheric delay	5%	2.5%	7.5%	2.5%

2. Using the time in seconds and adjusting the speed using the tropospheric delay, calculate the actual distance in meters between each satellite and the receiver.

   Use the box on the next page to show your working out.

Satellite A: ______________m

Satellite B: ______________m

Satellite C: ______________m

Satellite D: ______________m

Section 3: Scaling Distance

You will shortly be asked to mark the correct distances of the satellites on a grid (page 9).

The grid is drawn to scale 1cm = 4000m.

3. Using this information and your answers from Question 2 (page 6), work out the distance in cm from each satellite to the GPS receiver.

Satellite A: ______________**cm**

Satellite B: ______________**cm**

Satellite C: ______________**cm**

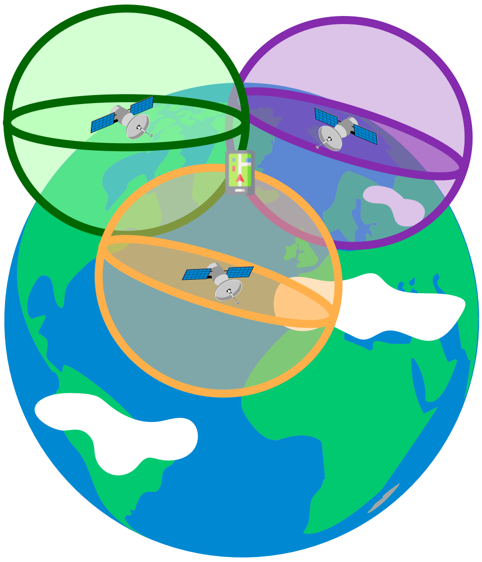
Satellite D: ______________**cm**

Section 4: Marking the Map

Position is calculated from distance measurements or "ranges" to satellites. We need at least 3 satellites to determine our position.

This is also known as **trilateration**.

Trilateration centres around finding your position on the Earth by knowing the location of 3 or more orbiting GPS satellites and the distance from those satellites to your location on Earth.

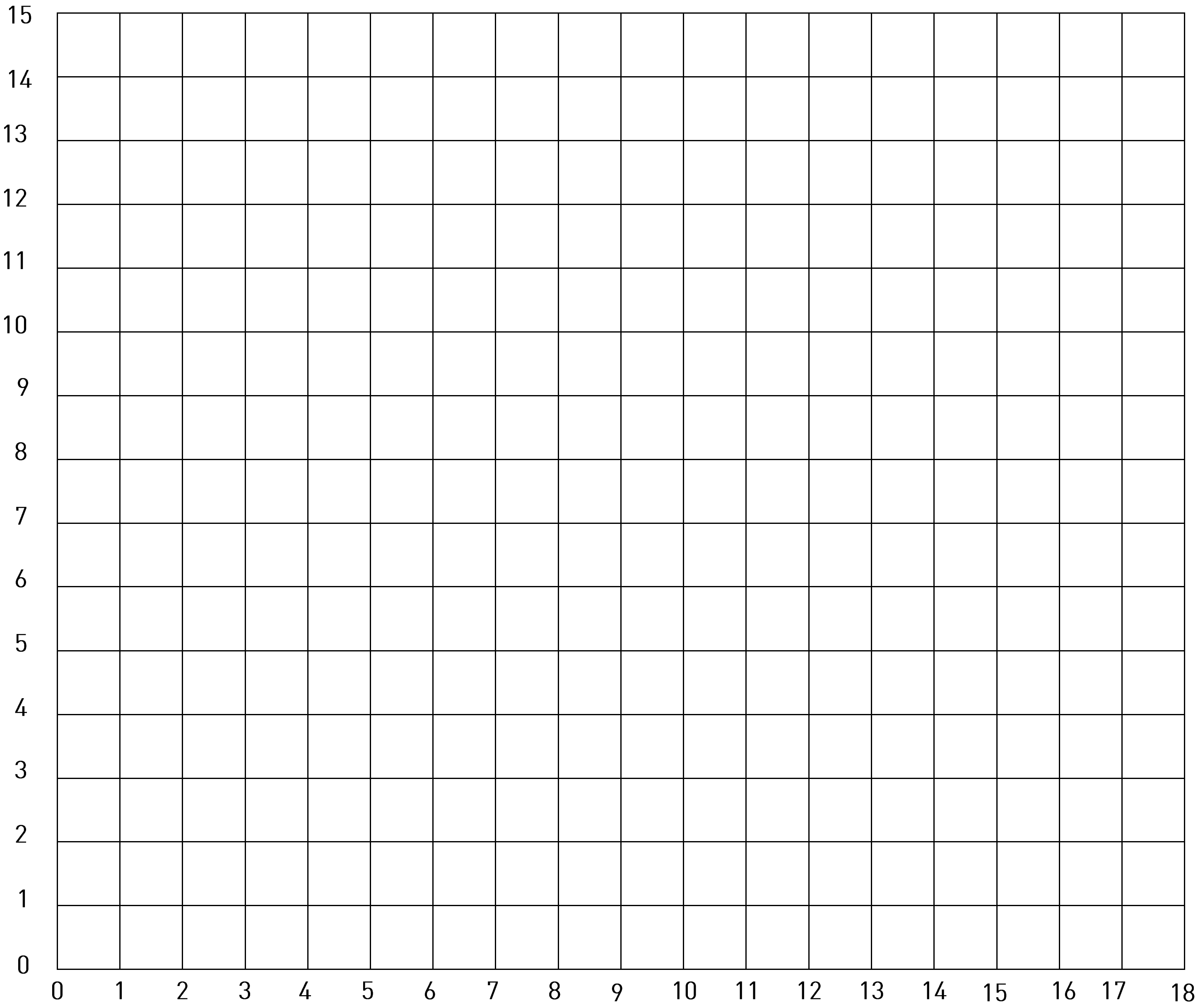


When calculating 3D position, we need 3 satellites to work out the X Y Z position of our receiver, but the receiver clock is poor compared to the atomic clock in the satellite. As the whole system is based on time, one more satellite is needed to determine the receiver clock bias, therefore we need at least 4 satellites.

4. Using the distances in cm you have calculated in Question 3 (page 7) and the coordinates given below, locate the GPS receiver on the grid on the next page using a pair of compasses and a ruler.

   The location of the GPS receiver is found where all the boundaries cross.

	Satellite A	Satellite B	Satellite C	Satellite D
Coordinates	2, 5	7, 10	11, 7	15, 8



Grid drawn to scale 1cm = 4000m

Section 5: Summary

In this activity you have learned how Global Positioning System (GPS) location is calculated using satellites which orbit the Earth.

First, you learned that those satellites transmit a unique signal that allow GPS devices to compute the precise location of the satellite. The satellite sends out the signal and records what time is sent it at, then the receiver records what time it receives the signal, then calculates the difference between the time the signal was sent from the satellite and time it was received.

Using the relationship *Distance = Speed × Time*, you worked out the distances from the satellites to the receiver by converting time to distance.

Then you calculated the actual distance between each satellite and the receiver, taking into account tropospheric delay.

Then you converted real life distances using a given scale and located an unknown point using loci and constructions.

If you enjoy using maths to solve problems, you might enjoy being a Geospatial Engineer!

There are more than 100 varied career roles across the surveying profession in three main sectors: construction and infrastructure, property, and land. Take a look at some examples below:

Land Surveyors use GPS to measure, survey, monitor and map the shape of the land very accurately for instance for construction projects

£20,000 - £70,000

UAV Surveyors use drones to capture aerial imagery and Lidar data to complete a survey of the area and processes the geospatial data to provide a map or 3D model

£25,000 - £45,000

Data Scientists analyse and process digital data into meaningful information that organisations can use to improve their businesses, such as maps

£25,000 – £100,000

To find out more about Geospatial Engineering, visit www.geospatialUK.org.

Answers

1. A) 23160 m

   B) 30900 m

   C) 25950 m

   D) 39900 m

2. A) 22002 m

   B) 30127 m

   C) 24003 m

   D) 38902 m

3. A) 5.5 cm

   B) 7.5 cm

   C) 6.0 cm

   D) 9.7 cm

4. 7, 2.5

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This activity was created by Newcastle University on behalf of Geospatial UK.

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For more resources or activities, visit

www.GeospatialUK.org

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